UUM 535
Engineering Mathematics
Fall 2017-2018

Asst. Prof. N. Kemal Ure
Aerospace Research Center
Istanbul Technical University

Lecture Times and Office Hours

Time: Fridays 09:30-12:30
Place: Faculty of Aeronautics and Astronautics, D-109
Additional Lecture Times and Place: Fridays 14:00-17:00, Place TBA
Prof. Ure Office Hours: Thursdays 13:00-14:00
Teaching Assistants: Mehmet Ugur Akcal (m.ugur.akcal@gmail.com) and Ahmed Farabi Tarhan (farabiahmed@gmail.com)

Course Objectives

- To provide students with a deep understanding of advanced mathematics on both theoretical and applied level.
- To present core theorems and applications of linear algebra, multivariable calculus and complex analysis.
- To provide students with the mathematical tools required to do research at the MSc and PhD level.

Prerequisites

An undergraduate level exposition to calculus and linear algebra is sufficient.

Lecture Topics

Introductory Lectures

- **Lecture 0:** Class Logistics
- **Lecture 1:** Foundations
  - Proof Techniques
  - Real and Complex Numbers
  - Field Axioms
  - Vector Spaces and Subspaces

Part I: Linear Algebra

- **Lecture 2:** Finite Dimensional Vector Spaces
  - Linear Independence
  - Span, Basis and Dimension
- **Lecture 3:** Linear Mappings

  - Null Spaces and Ranges
  - Matrices
  - Invertibility
  - Quotient and Dual Spaces
- **Lecture 4:** Eigenvalues and Invariant Subspaces
  - Polynomials and Fundamental Theorem of Algebra
  - Invariant Subspaces
  - Upper Triangular Matrices
  - Eigenspaces and Diagonal Matrices
- **Lecture 5:** Inner Product Spaces
  - Inner Products and Norms
  - Orthonormal Bases
  - Applications to Least Squares Optimization Problems
- **Lecture 6:** Operator Theory
  - Self-Adjoint and Normal Operators
  - Spectral Theorem
  - Positive Definite Matrices and Quadratic Forms
  - Singular Value Decomposition
  - Applications to Data Analysis Problems
- **Lecture 7:** Decompositions of Operators
  - Generalized Eigenvectors
  - Characteristic and Minimal Polynomials
  - Jordan Form
  - Trace of an Operator
  - Applications to Linear Control Problems
- **Lecture 8:** Trace and Determinant
  - Trace and Its Properties
  - Determinant and Its Properties

Part II: Multivariable Calculus

- **Lecture 9:** Continuity of Multivariable Functions
  - Basic theorem of Single Variable Calculus
  - Basic Topology
  - Sequences and Heine-Borel Theorem
  - Continuous Multivariable Functions
- **Lecture 10:** Derivative of Multivariable Functions
  - Bachmann-Landau Notation and Order Symbols
  - Derivative as a Linear Mapping
  - Chain and Composition Rules
  - Partial Derivatives and Gradient
  - Taylor's theorem for multivariable functions
  - Applications to Unconstrained Optimization
- **Lecture 11:** Inverse and Implicit Functions
  - Inverse Function Theorem
  - Implicit Function Theorem
  - Applications to Constrained Optimization
- **Lecture 12:** Multivariable Integration
  - Boxes, Partitions and Riemann Sums
  - Integrability Criteria
  - Integration Over Non-Boxes
  - Fubini's Theorem
  - Change of Variable Theorem
- **Lecture 13:** Parametrized Curves and Vector Analysis
  - Parametrized Curves
  - Curvature and Torsion
  - Flow and Flux Integrals
  - Divergence and Curl
  - Applications to Fluid Mechanics
- **Lecture 14:** Manifolds and Differential Forms
  - Algebra of Forms
  - Closed Forms, Exact Forms and Homotopy
  - Cubes and Chains
  - Integration on Manifolds
  - Generalized Fundamental Theorem of Calculus
  - Classical Theorems of Green, Gauss and Stokes

- Applications to Aerodynamics and Propulsion

Part III: Complex Analysis

- **Lecture 15:** Analytic Functions
  - Polar Representation and Roots of Complex Numbers
  - Elementary Complex Functions
  - Continuous Complex Functions
  - Basic Properties of Analytic Functions
  - Differentiation of Elementary Functions
  - Cauchy-Riemann Equations
- **Lecture 16:** Cauchy's Theorem
  - Contour Integrals
  - Cauchy's Theorem
  - Cauchy's Integral Formula
  - Harmonic Functions
- **Lecture 17:** Complex Series
  - Uniform Convergence
  - Power Series Expansion of Analytic Functions
  - Laurent Series and Classification of Singularities
- **Lecture 18:** Calculus of Residues
  - Residue Theorem
  - Evaluation of Definite Integrals
  - Infinite Series and Partial-Fraction Expansions
- **Lecture 19:** Conformal Mappings
  - Basic Theory of Conformal Mappings
  - Fractional Linear Transformations
  - Schwarz-Christoffel Transformations
  - Applications to Heat Transfer, Electrostatics and Hydrodynamics
- **Lecture 20:** Analytic Continuation
  - Riemann Surfaces
  - Asymptotic Methods
  - Gamma Function
  - Stirling's Formula and Bessel Functions
- **Lecture 21:** Laplace Transform
  - Basic Properties of Laplace Transforms
  - Complex Inversion Formula
  - Gamma Function
  - Applications to Ordinary Differential Equations

Textbooks

Theory Focused Textbooks:

- *Axler, S. J., Linear Algebra Done Right, 3rd Edition, Springer, 2015.*
- *Shurman, J. M., Calculus and Analysis in Euclidean Space. Springer, 2016.*
- *Marsden, J., Basic Complex Analysis., 3rd Edition. W. H. Freeman: New York, 1990.*

Application Focused Textbooks:

- *Strang, G., Linear Algebra and Its Applications 4th Edition, Cengage Learning, 2006*
- *Kreyszig, E., Advanced Engineering Mathematics, Wiley & Sons, 2010.*
- *Jeffrey, A., Advanced Engineering Mathematics, Academic Press, 2001.*

Grading

- Problem Sets 50%
- Take-home Midterm 20%
- Take-home Final Exam 30%

Grading Policies

- Cheating is strictly monitored and the penalty is -100 (minus hundred) points per assignment.
- Late assignments get -30 (minus thirty) points for each day after the deadline.
- No team work is allowed on problem sets and exams.
- Reports typeset with LaTeXget +10 bonus points.
- Each problem set will also contain a bonus problem, which will earn the student +10 bonus points if solved correctly.

Classroom Policies

- No attendance is required.
- Coming late to the class is tolerated.
- Bringing computers to the class is welcome.
- Unregistered listeners are welcome.
- Interacting with the the instructor is strongly recommended.
- Due to enormous content of the class, an additional three hour lecture will be conducted on every Friday (at 14:00-17:00, after the main lecture). This extra lecture will be videotaped and released to students.
- Extra problem solving sessions (date and place TBA) will be conducted by TA Ugur Akcal.